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Guide to Solving Trinomial Quadratic Equations

This guide uses 3 methods of solving equations: factoring to binomials, factoring through grouping, and the quadratic formula. These methods can be used to solve for the roots, zeros, and x-intercepts of a quadratic equation. The first method to try is factoring into the product of two binomials, if that cannot work, then try factoring through grouping, and if either one will not work, then use the quadratic formula as a sure-shot way of solving the problem. The goal of this packet is to guide you on how to perform these methods. It will benefit you to practice them until you can do these by heart. Before you can use any of these methods, it is very helpful to know how to read the signs in the problem to determine the factors you will use to solve it.

Read the Signs

To solve for the roots, zeros, and x-intercepts of quadratic equations, the problem must be set to zero. For example, the general form: $Ax^2 + Bx + C = 0$, is set to zero. First look at the right most sign in front of the C. If there is a "+", then it means that both signs when factoring will be the same as the sign on the left of B equation. For example, (+)(+) or (–)(–). If the sign is a "–" next to the C, then it means that signs when factoring are going to be different. For example, (+)(–) or (–)(+). Second, when reading the sign on the left (next to the B) it means that the largest factor of A*C is going to have the same sign in front of B.

Here are some examples of reading the signs in general factoring:

1st Right Side (in front of C):	**2nd** Left Side (in front of B):
+ = Same	+ = Largest factor is positive
– = Different	– = Largest factor is negative

Sign examples

$x^2 + 3x + 2 = 0$
$(x + 1)(x + 2) = 0$
Both signs are the same and are both positive.

$x^2 + x - 12 = 0$
$(x + 4)(x - 3) = 0$
Both signs are different and the largest factor (4) is positive.

$x^2 - 4x - 12 = 0$
$(x - 6)(x + 2) = 0$
Both signs are different and the largest factor (6) is negative.

$x^2 - 11x + 18 = 0$
$(x - 9)(x - 2) = 0$
Both signs are the same and are both negative.

Method Hierarchy

1st Factoring to Binomials

2nd Factoring Through Grouping

3rd Quadratic Formula

Factoring to Binomials

The first method to try is factoring the trinomial into a product of two binomials. You can use this method when there is no number in front of the x^2. For example, if you see: $x^2 - 5x + 6 = 0$, there is no number in front of x^2. It is understood to be a 1 in front of it. If there is a number in front of the x^2, check to see if you can factor out a greatest common factor (GCF). For example, if you see: $2x^2 - 10x + 12 = 0$, you can factor out the GCF of 2, which makes the equation become $2(x^2 - 5x + 6) = 0$ and then you can continue to factor the trinomial. ****If there is a number in front of the x^2 and a GCF cannot be factored, then try factoring through grouping.**** Here is an example of general factoring with the extra step of a GCF.

Example:

$2x^2 - 10x + 12 = 0$	Check for a GCF.
$2(x^2 - 5x + 6) = 0$	$Ax^2 + Bx + C = 0$ **A = 1, B = -5, and C = 6**
2**[**(x _)(x _)**]** = 0	Place parenthesis and x's, leaving the GCF on the outside and some space _ to plug in your factors. Next **Read the Signs**.
2**[**(x –)(x –)**]** = 0	Then determine your factors from **A*C**
A*C = 1 * 6 = 6	What are 2 factors of **A*C** (6) that will also sum up to **B** (-5)?
6 /\\ -1 * -6 & -1 + -6 = -7 -2 * -3 & -2 + -3 = -5	-2 * -3 = 6 and the sum of the factors is -5 which equals B. Place the factors in the parenthesis.
2**[**(x – 3)(x – 2)**]** = 0	Set the factored products equal to zero.
2 = 0 ; x – 3 = 0 ; x – 2 = 0	Solve for x.
$2 \neq 0$; **x = 3 ; x = 2**	The solutions are 3 and 2.

<u>2nd Factoring Through Grouping</u>

3rd Quadratic Formula

Factoring Through Grouping

The method of factoring through grouping solves a trinomial quadratic equation practically the same way as factoring it into a product of binomials. The only difference between grouping and the previous method is that grouping is <u>performed</u> slightly different. Grouping is used when you have four or more terms. In terms of trinomials, we can group by making the trinomial into a quadranomial. This can be done when there <u>is</u> a number in front of the x^2 <u>and</u> a GCF cannot be factored from each term. Here is an example of grouping in the works:

Example:

$2x^2 - 7x + 6 = 0$	There <u>is</u> a number in front of the x^2 and a GCF cannot be factored from each term.
$(\qquad) + (\qquad) = 0$	Place in parenthesis and **you need the + in the middle.**
$(2x^2 \qquad) + (\qquad + 6) = 0$	Bring down the first and last terms.
$(2x^2 \quad x) + (\quad x + 6) = 0$	Place in the x variables and leave enough space for signs and factors.
$(2x^2 - \quad x) + (- \quad x + 6) = 0$	**Read the Signs** in the original equation and place them in, leaving space for factors in front of the x's.
A = 2, B = -7, and C = 6 **A*C** = 2 * 6 = 12	Determine the factors.
12 /\\ -1 * -12 & -1 + -12 = -13 -2 * -6 & -2 + -6 = -8 -3 * -4 & -3 + -4 = -7	What are 2 factors of **A*C** (12) that also sum up to **B** (-7)? -3 * -4 = 12 <u>and</u> the sum of the factors equals -7 which equals B. Place the factors in the parenthesis.
$(2x^2 - 4x) + (-3x + 6) = 0$	Factor common numbers and variables.
$2x(x - 2) + -3(x - 2) = 0$	**The inside of the parenthesis must match!**
$(2x - 3)(x - 2) = 0$	Group the outsides and the insides together.
$2x - 3 = 0$; $x - 2 = 0$	Set the products equal to zero and solve.
$\mathbf{x = \frac{3}{2}}$; $\mathbf{x = 2}$	The solutions are $\frac{3}{2}$ and 2.

3^{rd} Quadratic Formula

Quadratic Formula

The last method in this packet is the quadratic formula. This is the sure-shot way of finding out your solutions if you cannot factor or group or if you're just not sure how to solve using those two methods. One step that can save you several steps is using the **discriminate** to your advantage. It can tell you whether there is/are one, two, or no solutions.

The Discriminate

$\sqrt{B^2 - 4AC}$ The discriminate can tell you how many solutions you get for x. You will use the general form $Ax^2 + Bx + C = 0$. You'll plug in the numbers for A, B, and C in the discriminate and the number that is under the square root determines the number of solutions for the problem. For example:

$\sqrt{275}$
If you end up with a positive number, that means there are **2 solutions**.

$\sqrt{0}$
If you end up with a zero, that means there is only **1 solution**.

$\sqrt{-16}$
If you end up with a negative number, that means there are **no real solutions** and 2 imaginary solutions.

Now let's say you were to solve for x for the given equation $36x^2 - 84x + 4 = 0$. The GCF that can be factored out is 2 and will help a little bit. Then, use the quadratic formula.

Quadratic Formula $x = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

Example:

$36x^2 - 84x + 4 = 0$

$4(9x^2 - 21x + 1) = 0$

A = 9, B = -21, and C = 1
Plug in the numbers.

$$x = \frac{-(-21) \pm \sqrt{(-21)^2 - 4(9)(1)}}{2(9)}$$

Simplify.

$$x = \frac{21 \pm \sqrt{441 - 36}}{18} = \frac{21 \pm \sqrt{405}}{18} = \frac{21 \pm 9\sqrt{5}}{18} = \frac{7 \pm 3\sqrt{5}}{6}$$

The solutions are x = $\dfrac{7 + 3\sqrt{5}}{6}$ **and x =** $\dfrac{7 - 3\sqrt{5}}{6}$

**Tip: If you your solutions completely simplify to whole numbers and/or fractions, that is to say, there are no squareroots as part of your final answers after using the quadratic formula, then it means that you could have use the previous two methods.