Graphing Rational Functions

I. Find the vertical and horizontal asymptotes of the function.

A. To find the *vertical* asymptotes of a rational function, set the denominator equal to zero and solve.

Example 1: $f(x) = \frac{2}{x-3}$

Since x - 3 = 0 when x = 3,
the vertical asymptote is at x = 3.

Example 2: $f(x) = \frac{x}{x^2+9}$

Since $x^2 + 9 = 0$ has no real root,
there is no vertical asymptote.

Example 3: $f(x) = \frac{x-2}{x^2+x-2}$

$x^2 + x - 2 = (x + 2)(x - 1) = 0$
when x = -2 or x = 1,
vertical asymptotes are at x = -2 and x = 1.

B. To find the *horizontal* asymptotes, compare the largest exponents of the numerator and the denominator.

1. **If the denominator has the higher exponent, then y = 0 is a horizontal asymptote.**
2. **If the numerator has the higher exponent, then there is no horizontal asymptote.**
3. **When the largest exponents are equal, then the horizontal asymptote is y = a/b where a and b are the leading coefficients as in the function below:**

$$f(x) = \frac{ax^n + \ldots}{bx^n + \ldots}$$

Example 1:

$f(x) = \frac{x-2}{x^2+x-2}$

Since the denominator has the higher exponent,
y = 0 is a horizontal asymptote.

Example 2:

$f(x) = \frac{x^3}{x^2-1}$

Since the numerator has the higher exponent,
there is no horizontal asymptote.

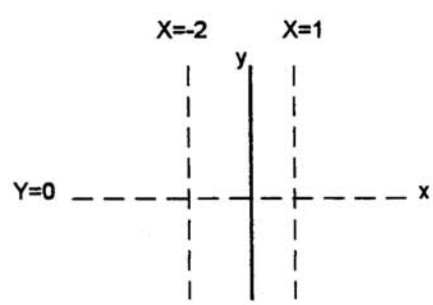
Example 3:

$f(x) = \frac{10x^2+x+2}{5x^2+8x+1}$ *Since the highest exponents in the numerator and the denominator are equal,* ***y = 10/5 = 2 is a horizontal asymptote.***

C. Graph the asymptotes as dashed lines.

Example:

Vertical asymptotes at x = -2
and x = 1
Horizontal asymptote at y = 0.



II. Find and plot the x- and y- intercepts of the function.

A. To find the x-intercepts, set the numerator equal to zero.

Example:

$$f(x) = \frac{x-2}{x^2+x-2}$$

Set $x - 2 = 0$; $x = 2$.
The function has an x-intercept at x = 2.
NOTE: If the numerator is a constant, then there is no intercept.

B. To find y-intercept, let x = 0 and evaluate f(0).

Example:
in the function
just above

$$f(0) = \frac{(0)-2}{(0)^2+(0)-2} = \frac{-2}{-2}$$

The y-intercept is at y = 1.

III. Make a table of sample values near each vertical asymptote and plot these values.

Example:

same function

x	y
.4	-0.6
-3	-1.3
-2.5	-2.5

x	y
-1.5	2.8
-1	1.5
0	1
0.8	2.1

x	y
1.1	-3
1.5	-0.3
3	0.1

IV) Sketch the curve.

Use the asymptotes and plotted points as guides. Remember that the function never crosses a vertical asymptote, and that a part of the function may pass through the horizontal asymptote. The function must pass through the plotted points.

Example:

$$f(x) = \frac{x-2}{x^2+x-2}$$

Vertical asymptotes at x = -2 and x = 1.

Horizontal asymptote at y = 0.

X-intercept at x = 2.

Y-intercept at y = 1.