Graphing with Calculus

Using the first derivative:

From the first derivative, we can find any **critical points** the function might have and the function's **periods of increase and decrease.** We can also use information from the first derivative to determine whether or not any **local extrema** (min's or max's) occur at these critical points.

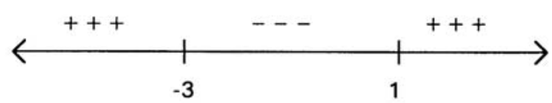
Example: $f(x) = x^3 + 3x^2 - 9x$

(A) To find the **critical points**, find all values of x where $f'(x)$ does not exist or is zero.

$$f'(x) = 3x^2 + 6x - 9 = 0$$
$$3(x^2 + 2x - 3) = 0$$
$$3(x-1)(x+3) = 0$$
$$x = 1 \quad \text{or} \quad x = -3$$

Since $f'(x)$ is defined for all x, -3 and 1 are the only *critical points.*

(B) Look at $f'(x)$ on a number line to determine **periods of increase and/or decrease.** Use the critical points as boundary numbers.



When $f'(x) > 0$, $f(x)$ is increasing. When $f'(x) < 0$, $f(x)$ is decreasing. Therefore (from the number line),

$f(x)$ is: *increasing* on $(-\infty, -3)$
decreasing on $(-3, 1)$
increasing on $(1, +\infty)$

(C) From the number line, determine whether any **local min's or max's** exist.

(1) Changing from increasing (positive) to decreasing (negative) means a local max occurs at that point.
(2) Changing from decreasing (negative) to increasing (positive) means a local min occurs at that point.
(3) No change means no local min or max.

In this example, at $x = -3$, $f(x)$ goes from increasing to decreasing.

Also at $x = 1$, $f(x)$ goes from decreasing to increasing.

Therefore: *local max* at $x = -3$
local min at $x = 1$

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